The secret structure of the gravitational vacuum[†]

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Abstract

We argue that the vacuum of quantum gravity must contain a hierarchical structure of correlations spanning all length scales. These correlated domains (called 'vecros') correspond to virtual fluctuations of black hole microstates. Larger fluctuations are suppressed by their larger action, but this suppression is offset by a correspondingly larger phase space of possible configurations. We give an explicit lattice model of these vecro fluctuations, noting how their distribution changes as the gravitational pull of a star becomes stronger. At the threshold of formation of a closed trapped surface, these virtual fluctuations transition into on-shell black hole microstates (fuzzballs). Fuzzballs radiate from their surface like normal bodies, resolving the information paradox. We also argue that any model without vecro-type extended vacuum correlations cannot resolve the paradox.

[†]Essay written for the Gravity Research Foundation 2024 Awards for Essays on Gravitation.
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Black holes have laid a trap for us. If we use our conventional understanding of general relativity and quantum theory, then we are led to a contradiction. To escape this trap we must perform a conjuring trick. In this essay we spell out what this trick must be. Like all conjuring tricks, once explained, it will hopefully appear simple and natural.

The trap:

Consider the gravitational collapse of a star. The curvatures are low everywhere at the point where the star falls through its horizon radius $r_h = 2GM$, so we are forced to agree that the star shrinks uneventfully through its horizon. New physics can certainly arise at the singularity where the star shrinks to planck size. But inside the horizon, light cones 'point inwards'. Thus if we accept that causality holds to leading order in any region with low curvature, then any new physics at the singularity cannot affect dynamics at horizon, at least to leading order. Furthermore, we can study the evolution using a set of 'good slices', which cover the horizon and exterior upto almost the endpoint of evaporation, but which do not approach the singularity and thus have low curvature everywhere. We are then forced to accept that semiclassical dynamics is valid throughout the evolution along these 'good slices'.

Now the trap has closed. Hawking [1] found that the quantum vacuum around the horizon is unstable to the creation of entangled particle pairs (b_i, c_i), whose state can be schematically modeled as

$$|\psi\rangle_{pair} = \frac{1}{\sqrt{2}} \left(|0\rangle_b |0\rangle_c + |1\rangle_b |1\rangle_c \right) \tag{1}$$

The b_i escape to infinity as 'Hawking radiation', leading to a monotonically growing entanglement of the radiation with the remaining hole. This creates a a sharp conflict at the endpoint of evaporation. Hawking's argument was made into a rigorous result by the small corrections theorem [2]: Any small correction to semiclassical dynamics will not remove the troublesome entanglement; we need an order *unity* correction.

The essential strength of the trap is that curvatures are low all through the evolution along the good slices. It is generally agreed that semiclassical physics must fail when curvatures reach planck scale, but to escape the trap, *we need a second mode of failure of semiclassical physics, one where curvatures are low everywhere.* What can give this second mode of failure?

To see what will *not* work, let us recall the traditional picture of quantum gravity in gently curved space. Fig.1(a) depicts a region of spacetime with a planck scale grid. We have quantum bits at each lattice site, interacting with the bits on neighboring sites. When spacetime stretches as in fig.1(b), new lattice sites get added to maintain a planck scale grid. The details of the bits and their interaction do not matter; any model in this universality class describes the same low energy semiclassical dynamics. This semiclassical dynamics fails only when curvatures reach planck scale, and so it does not fail along the good slicing of the black hole.

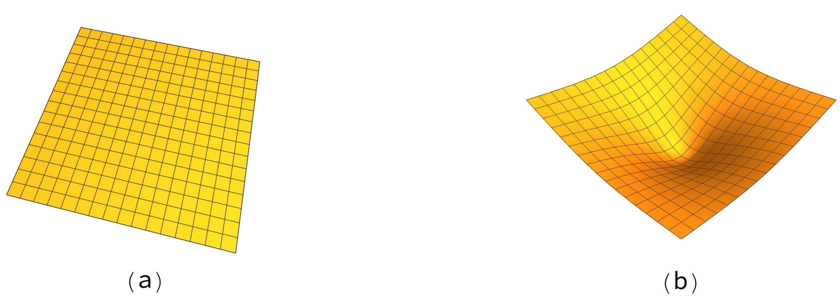

Figure 1: (a) Traditional models of quantum gravity assume a complicated structure at the planck scale, but standard low energy effective field theory at larger length scales in any region of gently curved spacetime. (b) As space stretches, we add new points to the grid, maintaining a planck scale lattice.

The role of vacuum fluctuations:

To see how to escape the trap, we first recall a somewhat similar conflict created by

Bekenstein's work of 1972. Bekenstein had argued that black holes have an entropy $S_{bek} \sim A/G$ [3]. Thermodynamics then implies that the hole must radiate at a temperature determined by $TdS_{bek} = dE$. But light cones 'point inwards' inside the horizon, so nothing should emerge from the region $r < 2GM$ to the outside. It seems that the hole cannot radiate, creating a conflict with thermodynamics.

As Hawking showed, once quantum field theory effects are taken into account, black holes do radiate in beautiful agreement with thermodynamics. But we can ask: How do the Hawking quanta magically appear from empty space?

The answer to this is of course well known. The vacuum is not empty to begin with; it is full of virtual particle-antiparticle pairs. The collapse of a star distorts spacetime, and the new geometry has a vacuum with a different set of virtual pairs. The difference between the old vacuum and the new one shows up as real (i..e not virtual) pairs; these are the quanta in (1) giving Hawking radiation. If we had failed to recognize the existence of these virtual fluctuations in the vacuum wavefunctional, then we would miss the phenomenon of Hawking radiation, and fail to get consistency with thermodynamics.

This history is useful, because in this essay we will argue the following. *We can escape the trap of the information paradox only if we assume that the gravitational vacuum has a much richer class of vacuum fluctuations than the particle-antiparticle pairs mentioned above.* We will give an explicit lattice model for these fluctuations, and show how they destroy semiclassical evolution in situations where a closed trapped surface forms.

To understand the full set of fluctuations of the quantum gravity vacuum, we must first understand the states of the black hole that account for its entropy

$$S_{bek} = \frac{A}{4G} \tag{2}$$

In string theory, several classes of microstates of black holes have been explicitly con-

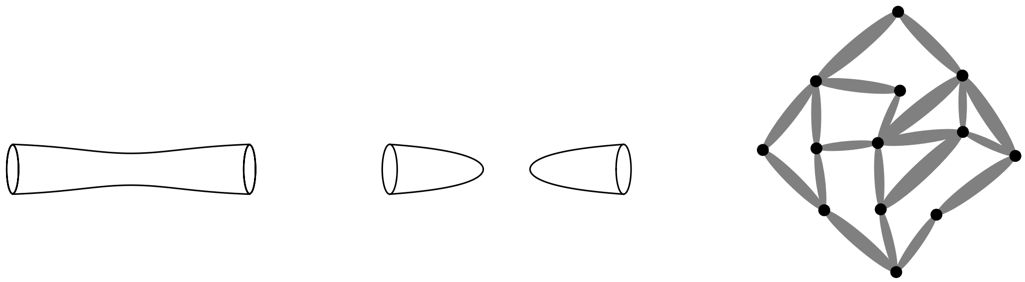

Figure 2: (a) Small deformations of compact directions give scalar fields. (b) Larger deformations create topological soliton like Kaluza-Klein monopoles. (c) Black hole microstates are 'fuzzballls': complicated bound sets of such solitonic objects.

structed, and in each case it is found that they have the structure of a *fuzzball*, which is like a normal body with no horizon [4]. The fuzzball conjecture states that all microstates will be of this form, with the surface of the generic fuzzball expected to be order planck distance outside the horizon radius [5].

To understand the structure of fuzzball states, first recall how usual scalar fields arise in string theory. 3+1 dimensional gravity is obtained when we compactify 6 directions to small circles. By the usual picture of Kaluza-Klein reduction, a scalar quantum in the 3+1 theory is obtained as a small deformation of the radius of a compact direction, depicted in fig.2(a). But we can also have a large deformation, depicted in fig.2(b), which changes the local topology of space: The full manifold is no longer a product of 3+1 spacetime with a compact manifold $\mathcal{M}_6$. In all cases where black hole microstates have been constructed, they are non-product manifolds of this kind: there is a complicated structure of KK-monpoles and antimonopoles, with gauge-field fluxes on the topological spheres connecting the monopoles centers. In [6, 7] it was shown how such non-product spacetimes invalidate the usual no-hair arguments and allow star-like structures to replace the traditional hole. The fuzzball are 'extended' objects which are very 'compression-resistant' due to their planck scale microstructure.

If fuzzballs like fig.2(c) exist as real objects for any mass M, then the vacuum must contain virtual fluctuations of these objects. The probability of such a fluctuation is estimated by $P = |A|^2$, with $A \sim Exp[-S_{grav}]$; here S_{grav} is the gravitational action to create the configuration. Setting all length scales as order $\sim GM$, we find that

$$S_{grav} \sim \frac{1}{G} \int \mathcal{R}\sqrt{-g}\, d^4x \sim GM^2 \sim \left(\frac{M}{m_p}\right)^2 \tag{3}$$

As expected, $P \ll 1$ for $M \gg m_p$. But this smallness is offset by the very large *degeneracy* of fuzzball states of mass M [8]

$$\mathcal{N} \sim e^{S_{bek}} \sim e^{\frac{A}{4G}} \sim e^{4\pi GM^2} = e^{4\pi\left(\frac{M}{m_p}\right)^2} \tag{4}$$

Thus we can have

$$P\mathcal{N} \sim 1 \tag{5}$$

Thus the virtual fluctuations of black hole microstates form an important component of the gravitational vacuum for all masses $0 < M < \infty$. We call these fluctuations *vecros*: "virtual extended compression-resistant objects" [9].

The conjuring trick we need:

To escape the trap, we must do the following seemingly impossible things:

(1) In the traditional picture of fig.1, *nothing* new happens as a star approaches the black hole threshold. In particular, local physics cannot determine if a horizon is about to form. But we *must* find something that becomes important at the black hole threshold!

(2) Since semiclassical physics is expected to be good until this threshold, whatever new effect we find for (1) must be an allowed feature of semiclassical dynamics.

(3) As we cross the black hole threshold, the quantum state should somehow transition to a fuzzball state of the kind depicted in fig.2(c).

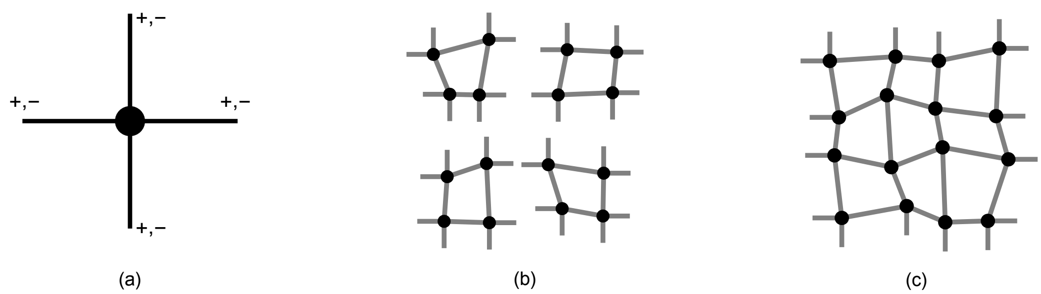

Figure 3: (a) A virtual topological excitation of the type in fig.2(b) is represented by a dot. (b) Such virtual excitations (vecros) can link together to make extended structures. (c) All sizes of vecros are relevant in the vacuum wavefunctional.

The way our model will escape the trap is as follows:

(1') The distribution of virtual fluctuations of black hole microstates (vecros) will change as we approach the black hole threshold, peaking at vecros with radius $r \approx 2GM$ at the threshold. The extended nature of the vecro fluctuations allows them to respond to the creation of a closed trapped surface: inside such a surface, the inward pointing structure of light cones squeezes the vecros to make them more compact.

(2') This change in the vecro distribution alters the vacuum of low energy modes from the Unruh state to the Boulware state. SInce the latter is also an allowed state of the semiclassical theory, we satisfy the requirement (2).

(3') As we cross the black hole threshold, the virtual vecro configurations transition to on-shell fuzzballs. Local energy balance is maintained because the Boulware state has negative Casimir energy, while the fuzzball structure has positive energy.

The model:

We now give an explicit lattice model to capture the intuitive argument in (5).

(i) We still have the mesh of fig.1 describing the fabric of spacetime, but in addition we can have local fluctuations at the lattice sites that are like the bubbles of fig.2(b). Since such bubbles can link together (fig.2(c)), we depict such a bubble in fig.3(a) by a

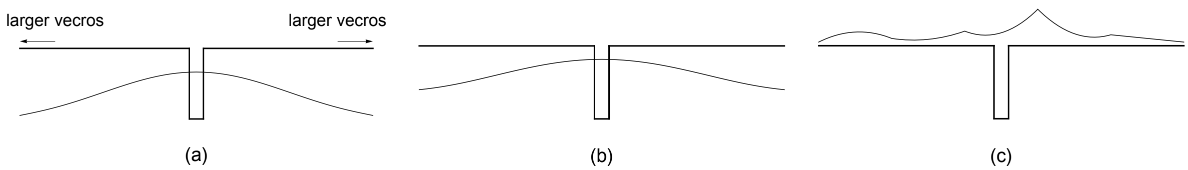

Figure 4: (a) The vecro distribution function in the vacuum. (b) The distribution shifts towards larger vecros under the gravitational attraction of a star. (c) Above the black hole threshold, the 'under the barrier' virtual wavefunction transitions to 'over the barrier' fuzzball states.

dot with 4 bonds sticking out from it. These bonds each carry a spin index $\pm$. There is a large phase space of virtual bubble fluctuations of this type, since the bonds on each bubble can have several choices of spins at their ends. But note that there is no entropy associated to such fluctuations; the different spin configurations have a fixed relative amplitude, giving the overall vacuum state $|\Psi_0\rangle$. This is similar to the usual fluctuations of particle-antiparticle pairs in the vacuum, which also do not imply any entropy for the vacuum; the pairs come with fixed relative amplitudes to yield the unique minimize energy vacuum state $|\Psi_0\rangle$.

(ii) If the bubble fluctuations occur on adjacent lattice sites, then their bonds can link to make a larger structure, as in fig.3(b). Such an extended fluctuation is a model of our 'vecro'. Since there are fewer open bonds when bonds link up, the phase space in fig.3(b) is smaller than the phase space where all bubbles were of the type in fig.3(a). But the energy of the configuration is *lowered* by this joining of bonds, making the larger vecro fluctuations as relevant as the smaller ones in the vacuum wavefunctional $|\Psi_0\rangle$. Similarly, we have a part of the vacuum wavefunctional with even larger correlated structures (fig.3(c)). This hierarchical clustering of virtual 'extended' structures models the intuition of eq.(5) that we have virtual fluctuations of extended objects at all scales in the vacuum wavefunctional. Each linked cluster in fig.3 is a 'vecro'.

(iii) Note that a quantum moving through spacetime does not 'bump into' or 'scatter off' virtual fluctuations, as long as the vacuum has approximate local translational in-

(a) (b)

Figure 5: (a) Vecro fluctuations in the vacuum. (b) Under the attraction of a central mass (the black annulus), the vecro distribution shifts towards larger vecros.

variance. Such fluctuations deform appropriately when a quantum reaches their location and relax to their original form when the quantum has passed through. Our string theory spacetime had 6 compact circles, and scalar fields in the spacetime were described by small fluctuations in the radii $R_i(x), i = 1, \dots 6$ of these circles (fig.2(a)). The vecros configurations depend on the R_i, and deform accordingly on passage of a quantum. But the only low energy degrees of freedom that can be excited at this stage are the Kaluza-Klein fields arising from the compactification, since the vecro distribution function in the present situation is determined by these low energy fields and is not an independent variable.

(iv) We need a schematic picture to describe the relative amplitudes of vecros of different sizes in our wavefunctional. Recall that in a 1-dimensional potential well, the virtual fluctuation is the part of the wavefunction that is 'under the barrier'. In fig.4(a) we draw such a potential well and let the part of the wavefunction under the barrier denote the 'vecro distribution function'. As mentioned above, at this stage the shape of this vecro distribution function is fixed to a unique shape that minimizes the energy of the state, so this part of the wavefunctional does not describe any independent excitations.

(v) In fig.5 we describe what happens when instead of the vacuum, we have a dense mass like a star in our spacetime. The vecro configurations of the vacuum in fig.5(a) get attracted and squeezed if we place a heavy object at the center as in fig.5(b). This squeezing causes the smaller vecros to join up into larger ones. We depict this change schematically in fig.4(b): under the attraction of a heavy central mass, the vecro distribution function shifts towards larger vecros. This shift towards larger vecros continues till the mass of the star reaches the black hole threshold.

(vi) What happens if we add further energy to the star so that it crosses the black hole threshold? The gravitational pull of the star has already caused the vecros to join up till they are almost all of radius $r \approx 2GM$. As we cross the black hole threshold, the wavefunctional over the vecro configuration ceases to be 'under the barrier', and becomes an oscillatory function (fig.4(c)). This means that the relative amplitudes of different vecro configurations is not fixed; instead we can have many different wavefunctions over the space of vecro configurations. If the wavefunction is taken to peak around a given configuration, then we get the corresponding fuzzball of fig.2(c). The fuzzball is not a virtual fluctuation: once the wavefunction becomes oscillatory we have real (i.e. on-shell) configurations. Since the space of fuzzball configurations is large, we now get a large number of allowed states, corresponding to the Bekenstein entropy (2).

(vii) Finally, we should ask what triggers the transition from virtual excitations (vecros) to on-shell fuzzballs exactly as we cross the black hole threshold. The vecro excitations relevant to our dynamics are extended structures that are at rest in the Schwarzschild frame. The gravitational pull in the Schwarzschild frame diverges as $r \to 2GM^+$, so the compression effect on vecros also diverges as we approach the horizon radius. Correspondingly, the distribution of vacuum fluctuations is not smooth in the limit $r \to 2GM^+$: the amplitude for a vecro fluctuation to exist diverges as the size of the vecro is taken towards $r = 2GM^+$. Because of this, in contrast to the situation in (iii) above, the spectrum of fluctuations is not locally translationally invariant near $r = 2GM$. Thus the local vacuum is not the Unruh vacuum (which *is* translationally invariant through $r = 2GM$) but a state closer to the Boulware vacuum. The Boulware vacuum has a negative Casimir energy, and this balances the positive energy of the fuzzballs which emerge from the vecro fluctuations as in (vi). Note that an initial Unruh horizon can form during rapid gravitational collapse, but the vecros of size $r \approx 2GM$ are static in the Schwarzschild frame and would therefore feel an Unruh acceleration temperature $\frac{1}{2\pi s}$ at a distance s outside the horizon. As $s \to 0$, this is a diverging excitation of the vecros, and alters their distribution to yield on-shell the fuzzballs mentioned above.

Summary:

Electrons and positrons exist as real particles, so the vacuum must contain corresponding *virtual* fluctuations. Recognizing this fact leads to the process of Hawking radiation. This process allows consistency with thermodynamics, but leads to the trap of the information paradox. We escape the trap if we recognize that black holes have a large number of microstates, and thus the vacuum must have corresponding virtual fluctuations. In string theory many of these microstates have been explicitly constructed and are found to have an extended structure with no horizon (fig.2(c)). The corresponding virtual fluctuations, called vecros, give the gravitational vacuum a secret structure where it has correlations at all length scales. In this article we have given an explicit lattice model of these correlations, where the gravitational attraction of a star leads to a squeezing of these vecros; this squeezing alters the vecro distributions and converts them to on-shell fuzzball states at the black hole threshold. (In [10] it was shown that such fuzzballs would naturally acquire the Hawking temperature $T_H = 1/8\pi GM$.) Since fuzzballs have no horizon and radiate from their surface like a normal body, we resolve the information paradox. It is crucial that vecro fluctuations are extended in size, so they are able to 'feel around' a region and respond to the formation of a closed trapped surface. By contrast, the traditional picture of the vacuum depicted in fig.1 sees nothing special upon horizon formation, and so cannot escape the trap of the paradox.

Acknowledgements

This work is supported in part by DOE grant DE-SC0011726. I would like to thank Robert Brandenberger, Emil Martinec and Madhur Mehta for several helpful discussions.

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