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Symmetry Breaking for Matter Coupled to Linearized Supergravity From the Perspective of the Current Supermultiplet

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ABSTRACT

We consider a generic supersymmetric matter theory coupled to linearized supergravity, and analyze scenarios for spontaneous symmetry breaking in terms of vacuum expectation values of components of the current supermultiplet. When the vacuum expectation of the energy momentum tensor is zero, but the scalar current or pseudoscalar current gets an expectation, evaluation of the gravitino self energy using the supersymmetry current algebra shows that there is an induced gravitino mass term. The structure of this term generalizes the supergravity action with cosmological constant to theories with CP violation. When the vacuum expectation of the energy momentum tensor is nonzero, supersymmetry is broken; requiring cancellation of the cosmological constant gives the corresponding generalized gravitino mass formula.

Supersymmetry, to be relevant to physics, must be broken, and mechanisms for supersymmetry breaking have been intensively studied. In this essay, we shall analyze scenarios for spontaneous symmetry breaking in locally supersymmetric theories by reference to the vacuum expectation values of the components of the current supermultiplet, through which a generic supersymmetric matter theory couples to linearized supergravity.

In linearized general relativity, the spacetime metric $g_{\mu\nu}$ deviates from the Minkowski metric $\eta_{\mu\nu}$ by a small perturbation $h_{\mu\nu}$,

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu} \quad , \tag{1}$$

with the proportionality constant κ related to Newton's constant G and the Planck mass $M_{\rm Planck}$ by

$$\kappa = (8\pi G)^{\frac{1}{2}} = M_{\rm Planck}^{-1} \quad . \tag{2}$$

In linearized supergravity, one adjoins to the spin 2 graviton field $h_{\mu\nu}$ a spin 3/2 Rarita-Schwinger Majorana field ψ_μ, which describes the fermionic gravitino partner of the graviton. A gravity supermultiplet, for which the supersymmetry algebra closes without use of the equations of motion, is obtained by adding auxiliary fields, consisting [1] of an axial vector b_μ, a scalar M, and a pseudoscalar N. The supersymmetry variations which close the supersymmetry algebra (with constant Grassmann supersymmetry parameter ϵ, and with $a \cdot c \equiv a_\mu c^\mu$) are

$$\begin{aligned}
\delta h_{\mu\nu} =& \frac{1}{2}\bar{\epsilon}(\gamma_\mu \psi_\nu + \gamma_\nu \psi_\mu) \quad , \\
\delta \psi_\mu =& [-\sigma^{\kappa\nu}\partial_\kappa h_{\nu\mu} - \frac{1}{3}\gamma_\mu(M + i\gamma_5 N) + (b_\mu - \frac{1}{3}\gamma_\mu \gamma \cdot b) i\gamma_5]\epsilon \quad , \\
\delta b_\mu =& \frac{3}{2} i\bar{\epsilon}\gamma_5 (R_\mu - \frac{1}{3}\gamma_\mu \gamma \cdot R) \ , \quad \delta M = -\frac{1}{2}\bar{\epsilon}\gamma \cdot R \ , \quad \delta N = -\frac{1}{2} i\bar{\epsilon}\gamma_5 \gamma \cdot R \quad .
\end{aligned} \tag{3}$$

The corresponding linearized supergravity action, which is invariant under these variations, is

$$S_{\text{grav}} = \int d^4x [E^{\mu\nu} h_{\mu\nu} - \frac{1}{2} \overline{\psi}_\mu R^\mu - \frac{1}{3}(M^2 + N^2 - b_\mu b^\mu)] \quad , \tag{4}$$

with $E^{\mu\nu}$ the linearized Einstein tensor and with $R^\nu = i\epsilon^{\nu\mu\kappa\rho}\gamma_5\gamma_\mu\partial_\kappa\psi_\rho$.

Linearized supergravity couples to supersymmetric matter through a real supermultiplet of currents [2], consisting of the energy momentum tensor $\theta^{\mu\nu}$, the supersymmetry current j_μ, an axial vector current $j_\mu^{(5)}$, a scalar density P, and a pseudoscalar density Q. These transform [3] under supersymmetry variations as

$$\begin{aligned}
\delta\theta^{\mu\nu} =& \frac{1}{4}\overline{\epsilon}(\sigma^{\kappa\mu}\partial_\kappa j^\nu + \sigma^{\kappa\nu}\partial_\kappa j^\mu) \quad , \\
\delta j_\mu =& [2\gamma^\nu\theta_{\mu\nu} - i\gamma_5\gamma\cdot\partial j_\mu^{(5)} + i\gamma_5\gamma_\mu\partial\cdot j^{(5)} + \frac{1}{2}\epsilon_{\mu\nu\rho\kappa}\gamma^\nu\partial^\rho j^{\kappa(5)} + \frac{1}{3}\sigma_{\mu\nu}\partial^\nu(P + i\gamma_5 Q)]\epsilon \quad , \\
\delta j_\mu^{(5)} =& i\overline{\epsilon}\gamma_5 j_\mu - \frac{1}{3}i\overline{\epsilon}\gamma_5\gamma_\mu\gamma\cdot j \ , \quad \delta P = \overline{\epsilon}\gamma\cdot j \ , \quad \delta Q = i\overline{\epsilon}\gamma_5\gamma\cdot j \quad .
\end{aligned} \tag{5}$$

The matter interaction action that is invariant under simultaneous supersymmetry variations of the gravity and current supermultiplets, and that gives the correct Newtonian static limit, is

$$S_{\text{int}} = \kappa \int d^4x [h_{\mu\nu}\theta^{\mu\nu} + \frac{1}{2}\overline{\psi}_\mu j^\mu - \frac{1}{2} b_\mu j^{\mu(5)} - \frac{1}{6}(MP + NQ)] \quad . \tag{6}$$

Since the auxiliary fields b_μ, M, and N enter with no differential operators acting on them, their equations of motion following from Eqs. (4) and (6) are the algebraic relations

$$b_\mu = \frac{3}{4}\kappa j_\mu^{(5)} \ , \quad M = -\frac{1}{4}\kappa P \ , \quad N = -\frac{1}{4}\kappa Q \quad . \tag{7}$$

Using Eq. (7), one can eliminate the auxiliary fields from the combined supergravity and interaction actions. As we have recently shown [3], by completing the square one can also eliminate the graviton and gravitino fields from the linearized theory. This gives the full effective action $S_{\rm eff}$ which describes the order κ^2 back reaction of supergravity on the matter sector,

$$
\begin{aligned}
S_{\rm eff} =&\kappa^2 \int d^4x \left[-\frac{3}{16} j^{(5)}_\mu j^{\mu(5)} + \frac{1}{48}(P^2+Q^2) \right] \\
&+\kappa^2 \int d^4x d^4y \left[\frac{1}{4}\theta^{\nu\tau}(x)(\eta_{\nu\alpha}\eta_{\tau\beta}+\eta_{\nu\beta}\eta_{\tau\alpha}-\eta_{\nu\tau}\eta_{\alpha\beta})\Delta_F(x-y)\theta^{\alpha\beta}(y) \right. \\
&\left. -\frac{1}{8}\bar{j}_\tau(x)\left(\eta^{\tau\nu}\gamma\cdot\partial_x+\frac{1}{2}\gamma^\tau\gamma\cdot\partial_x\gamma^\nu\right)\Delta_F(x-y)j_\nu(y)\right] \quad ,
\end{aligned}
\tag{8}
$$

with Δ_F the massless Feynman propagator

$$
\Delta_F(x-y) = \frac{1}{(2\pi)^4}\int d^4q \frac{e^{iq\cdot(x-y)}}{q^2-i0^+} \quad . \tag{9}
$$

Using conservation of the currents j_μ and $\theta_{\mu\nu}$, one can show that Eq. (8) is invariant under the supersymmetry transformation on the current supermultiplet given in Eq. (5).

To examine the implications of the above relations for spontaneous symmetry breaking, we take vacuum expectations (denoted by $\langle \quad \rangle$) of Eqs. (5) and (7). Because Lorentz invariance requires the vanishing of the vacuum expectations $\langle j^{(5)}_\mu \rangle$, $\langle j_\mu \rangle$, and $\langle b_\mu \rangle$, while $\langle \theta_{\mu\nu} \rangle$ can be proportional to the Minkowski metric $\eta_{\mu\nu}$, and so can be nonzero, Eq. (5) gives

$$
\begin{aligned}
\langle \delta\theta^{\mu\nu} \rangle =& \langle \delta j^{(5)}_\mu \rangle = \langle \delta P \rangle = \langle \delta Q \rangle = 0 \quad , \\
\langle \delta j_\mu \rangle =& [2\gamma^\nu \langle \theta_{\mu\nu} \rangle + \frac{1}{3}\sigma_{\mu\nu}\partial^\nu(\langle P \rangle + i\gamma_5 \langle Q \rangle)]\epsilon \quad ,
\end{aligned}
\tag{10}
$$

and Eq. (7) gives

$$
\langle M \rangle = -\frac{1}{4}\kappa\langle P \rangle \ , \quad \langle N \rangle = -\frac{1}{4}\kappa\langle Q \rangle \quad . \tag{11}
$$

Since $\langle P \rangle$, $\langle Q \rangle$ are coordinate independent by translation invariance, they do not contribute to the right hand side of Eq. (10). Hence Eq. (10) for $\langle \delta j_\mu \rangle$ simplifies to

$$
\langle \delta j_\mu \rangle = 2\gamma^\nu \langle \theta_{\mu\nu} \rangle \epsilon \quad . \tag{12}
$$

Let us first consider the case when $\langle\theta_{\mu\nu}\rangle = 0$. The set of expectations

$$\begin{aligned} \langle\theta_{\mu\nu}\rangle =&\langle j_\mu\rangle = \langle j_\mu^{(5)}\rangle = 0 \quad , \\ \langle P\rangle \neq& 0 \ , \quad \langle Q\rangle \neq 0 \quad , \end{aligned} \tag{13}$$

satisfy Eqs. (10) if we take the supersymmetry variations of the expectations to be $\delta\langle P\rangle = \langle\delta P\rangle = 0$, $\delta\langle Q\rangle = \langle\delta Q\rangle = 0$. Thus, the transformation properties of the supermultiplet of currents are preserved when the scalar current P and the pseudoscalar current Q develop nonzero vacuum expectations that are supersymmetry invariants.

Whether P and/or Q have nonzero expectations is a matter of detailed dynamics. An important case where $\langle P\rangle \neq 0$, but supersymmetry remains unbroken, is supersymmetric Yang-Mills theory. In this theory P is related to the gaugino density through the scalar component of the anomaly supermultiplet,

$$P = g^{-1}\beta(g)\overline{\chi}\chi \quad , \tag{14}$$

with g the Yang-Mills coupling, and hence P develops a nonzero expectation,

$$\langle P\rangle = g^{-1}\beta(g)\langle\overline{\chi}\chi\rangle \quad , \tag{15}$$

as a result of the formation [4] of a vacuum gaugino condensate.

When P and/or Q has a nonzero expectation, Eq. (8) implies a nonzero vacuum energy density (the negative of the vacuum action density) given by

$$\rho_{\rm VAC} = -\frac{\kappa^2}{48}(\langle P\rangle^2 + \langle Q\rangle^2) \quad . \tag{16}$$

There are two other places where effects arising from $\langle P\rangle$ and $\langle Q\rangle$ appear. First, from Eqs. (3) and (11), we see that the supersymmetry variation of the gravitino field receives a contribution from $\langle P\rangle$ and $\langle Q\rangle$ given by

$$\delta\psi_\mu = \frac{\kappa}{12}\gamma_\mu(\langle P\rangle + i\gamma_5\langle Q\rangle)\epsilon + \ldots \quad , \tag{17}$$

with ... denoting terms with expectation zero.

Second, $\langle P \rangle$ and $\langle Q \rangle$ contribute to the gravitino self energy. To order κ^2, the gravitino self energy induced by matter couplings is given by the action addition

$$\Delta S = i\frac{\kappa^2}{8}\int d^4x d^4y \overline{\psi}_{\mu A}(x)\langle T(j^\mu_A(x)\overline{j}^\rho_B(y))\rangle \psi_{\rho B}(y) \quad , \tag{18}$$

with A, B spinor indices. The action term involving no derivatives of the gravitino field is obtained by treating the gravitino field as a constant in Eq. (18), leading to

$$\Delta S \simeq i\frac{\kappa^2}{8}\int d^4x \overline{\psi}_{\mu A}(x)\langle K^{\mu\rho}_{AB}\rangle \psi_{\rho B}(x) \quad , \tag{19}$$

with the constant operator $K^{\mu\rho}_{AB}$ defined by

$$K^{\mu\rho}_{AB} \equiv \frac{\int d^4x \int d^4y T(j^\mu_A(x)\overline{j}^\rho_B(y))}{\int d^4x \ 1} \quad . \tag{20}$$

To evaluate $K^{\mu\rho}_{AB}$, we use current algebra methods, by expanding the identity

$$0 = \int d^4x d^4y \frac{\partial}{x^\theta}[x^\mu T(j^\theta_A(x)\overline{j}^\rho_B(y))] \quad , \tag{21}$$

giving

$$K^{\mu\rho}_{AB} = \frac{-\int d^4x d^4y x^\mu [T(\partial \cdot j_A(x)\overline{j}^\rho_B(y)) + \delta(x^0 - y^0)\{j^0_A(x), \overline{j}^\rho_B(y)\}]}{\int d^4x \ 1} \quad . \tag{22}$$

Using conservation of j^θ_A, together with the fact that j^0_A is the supersymmetry generator obeying

$$\overline{\epsilon}_A\{j^0_A(x), \overline{j}^\rho_B(y)\} = -i\delta^3(\vec{x} - \vec{y})\delta\overline{j}^\rho_B(y) \quad , \tag{23}$$

and using Eq. (5) to calculate the supersymmetry variation on the right hand side of Eq. (23), we get

$$\begin{aligned} K^{\mu\rho}_{AB} =& \frac{-\int d^4x d^4y x^\mu \delta^4(x-y)\left(\frac{-i}{3}\right)\sigma^{\tau\rho}_{AB}\frac{\partial}{y^\tau}[P(y) + i\gamma_5 Q(y)] + ...}{\int d^4x \ 1} \\ =& -\frac{i}{3}\sigma^{\mu\rho}_{AB}\frac{\int d^4x [P(x) + i\gamma_5 Q(x)] + ...}{\int d^4x \ 1} \quad . \end{aligned} \tag{24}$$

The terms denoted by ... do not contribute to the expectation, and so Eq. (24) implies

$$\langle K_{AB}^{\mu\rho}\rangle = -\frac{i}{3}\sigma_{AB}^{\mu\rho}(\langle P\rangle + i\gamma_5\langle Q\rangle) \quad , \tag{25}$$

which when substituted into Eq. (19) gives the gravitino mass term

$$\Delta S_{\text{mass}} = \frac{\kappa^2}{24}\int d^4x\overline{\psi}_\mu(x)(\langle P\rangle + i\gamma_5\langle Q\rangle)\sigma^{\mu\rho}\psi_\rho(x) \quad . \tag{26}$$

When the CP violating expectation $\langle Q\rangle$ is zero, Eqs. (16), (17), and (26) are respectively the vacuum energy density, the modified gravitino variation, and the gravitino mass term that enter into the extension [5] of supergravity to accommodate a nonvanishing cosmological constant, corresponding to supergravity in anti-de Sitter space [6]. When the expectation $\langle Q\rangle$ is nonzero, these equations give a generalized supergravity with cosmological constant, in which there is also a CP violating gravitino mass term.

This generalized supergravity is supersymmetric even beyond the linearized approximation. To see this, we make the polar decomposition

$$\langle P\rangle + i\gamma_5\langle Q\rangle = (\langle P\rangle^2 + \langle Q\rangle^2)^{\frac{1}{2}}e^{i\alpha\gamma_5} \ , \qquad \alpha = \arctan(\langle Q\rangle/\langle P\rangle) \quad . \tag{27}$$

and define new gamma matrices $\tilde{\gamma}_\mu = \gamma_\mu \exp(i\alpha\gamma_5) = \exp(-i\alpha\gamma_5/2)\gamma_\mu \exp(i\alpha\gamma_5/2)$, which obey the same identities as the γ_μ, as well as $\tilde{\gamma}_\mu\tilde{\gamma}_\nu = \gamma_\mu\gamma_\nu$. Since $\overline{\psi}$ contains a factor γ^0, we see that Eqs. (17) and (26) plus the gravitino kinetic term are equivalent to the theory with $\langle Q\rangle = 0$, with $\langle P\rangle$ replaced by $(\langle P\rangle^2 + \langle Q\rangle^2)^{\frac{1}{2}}$, and with all γ_μ replaced by the corresponding $\tilde{\gamma}_\mu$, to which the supersymmetry proofs of Refs. [5] apply.

Let us turn now to the generic case, in which the expectations $\langle\theta_{\mu\nu}\rangle$, $\langle P\rangle$, and $\langle Q\rangle$ are all nonzero. Using $\langle\theta_{\mu\nu}\rangle = \langle\theta_0^0\rangle\eta_{\mu\nu}$, Eq. (12) can be rewritten as

$$\langle\delta j_\mu\rangle = 2\gamma_\mu\langle\theta_0^0\rangle\epsilon \quad . \tag{28}$$

Since a nonzero value of $\langle j_\mu \rangle$ implies that supersymmetry is broken, we recover the usual criterion, that supersymmetry in the matter sector is broken if and only if the positive semidefinite matter vacuum energy density $\langle \theta^0_0 \rangle$ is nonzero. Adding $\langle \theta^0_0 \rangle$ to Eq. (16), the total vacuum energy density becomes

$$\rho_{\mathrm{VAC}} = \langle \theta^0_0 \rangle - \frac{\kappa^2}{48}(\langle P \rangle^2 + \langle Q \rangle^2) \quad . \tag{29}$$

Rewriting Eq. (26) as

$$\begin{aligned} \Delta S_{\mathrm{mass}} =& \frac{1}{2} m \int d^4x \overline{\psi}_\mu(x) \sigma^{\mu\rho} \psi_\rho(x) + \frac{1}{2} m' \int d^4x \overline{\psi}_\mu(x) i\gamma_5 \sigma^{\mu\rho} \psi_\rho(x) \quad , \\ m =& \frac{\kappa^2}{12} \langle P \rangle \ , \quad m' = \frac{\kappa^2}{12} \langle Q \rangle \quad , \end{aligned} \tag{30}$$

the condition for the vacuum energy density ρ_{VAC} of Eq. (29) to vanish by cancellation between the matter and supergravity contributions is

$$\kappa \left[\frac{\langle \theta^0_0 \rangle}{3} \right]^{\frac{1}{2}} = \frac{\kappa^2}{12} (\langle P \rangle^2 + \langle Q \rangle^2)^{\frac{1}{2}} = (m^2 + m'^{\,2})^{\frac{1}{2}} \quad . \tag{31}$$

We thus obtain a new derivation (when $\langle Q \rangle = m' = 0$) of the Deser-Zumino [7] formula for the gravitino mass, as well as its extension to the case when the CP violating expectation $\langle Q \rangle$ is nonzero.

To conclude, we have shown that by using the transformation properties of the current supermultiplet, one can analyze possibilities for supersymmetry breaking when supersymmetric matter is coupled to linearized supergravity. Nonlinear supergravity corrections to our results appear only at higher orders in the expansion in powers of κ. In addition to giving a compact current-algebraic derivation of the action for supergravity with a cosmological constant, and of the gravitino mass formula, our method generalizes these results to the case when the matter theory breaks CP invariance, allowing the expectation $\langle Q \rangle$ to be nonzero.

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2802. Our formulation is closest to that of Townsend. Note that $\sqrt{2}\kappa$ in Townsend's paper is our κ, and ϵ of Townsend's paper is our $\sqrt{2}\epsilon$, so that $\kappa\epsilon$ is the same in both.

[6] I wish to thank E. Witten for alerting me to the role of anti-de Sitter supergravity in this discussion.

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