SUPERGRAVITY THEORY BEYOND ELEVEN DIMENSIONS

WANG Mian

Henan Normal University

Xinxiang, Henan, China

Summary

We study supergravity theory in higher dimensional spaces. Beyond eleven dimensions, it is convinced that twelve dimensional space with signature (2,10) is the candidate for supergravity theory. In this space, there exists Majorana-Weyl spinor, then the theory contains no particle with spin $J > 2$. The action

$$I = \int d^{12}x\, \mathcal{L}$$

$$\begin{aligned}\mathcal{L} = & -\frac{e}{2}R - \frac{i}{2}e\bar{\psi}_M\Gamma^{MNP}D_N\psi_P - \frac{e}{48}F_{MNPQ}F^{MNPQ} - \frac{i}{2}e\bar{\lambda}\not{D}\lambda - \frac{e}{2}(\partial_M A)^2 - \frac{e}{2}(\partial_M B)^2 \\ & + \frac{e}{192}\left[\bar{\psi}_K\Gamma^{KLMNPQ}\psi_L + 12\bar{\psi}^M\Gamma^{NP}\psi^Q\right]F_{MNPQ} + \frac{i}{96}\sqrt{2}e\left[\bar{\lambda}\Gamma^{MNPQ}\Gamma^S\psi_S\right]F_{MNPQ} \\ & + \frac{e}{4}\bar{\lambda}\not{D}(A - i\bar{\Gamma}B)\Gamma^S\psi_S - \frac{e}{24^2}(\bar{\lambda}\Gamma^{MNPQ}\lambda)F_{MNPQ} + \text{four-fermion terms}\end{aligned}$$

is invariant under

$$\delta e_M^A = \frac{i}{2}\bar{\epsilon}\Gamma^A\psi_M, \qquad \delta A = \bar{\epsilon}\lambda, \qquad \delta B = i\bar{\epsilon}\bar{\Gamma}\lambda,$$

$$\delta\lambda = -i\not{D}(A + i\bar{\Gamma}B)\epsilon + \frac{\sqrt{2}}{24}\Gamma^{MNPQ}F_{MNPQ}\,\epsilon,$$

$$\delta\psi_M = D_M\epsilon + \frac{i}{288}\left(\Gamma^{PQRS}{}_M - 8\delta_M^P\Gamma^{QRS}\right)F_{PQRS}\,\epsilon,$$

$$\delta A_{MNP} = \bar{\epsilon}\Gamma_{MN}\psi_P + i\sqrt{2}\,\bar{\epsilon}\Gamma_{MNP}\lambda.$$

SUPERGRAVITY THEORY BEYOND 11 DIMENSIONS

WANG Mian

Henan Normal University, Xinxiang, Henan, China

Supergravity theory beyond 11 dimensions is investigated. It is shown that in a 12-dimensional space with signature (2,10) there exists Majonara-Weyl spinor, the number of independent components of such a spinor is 16 as a Majorana spinor in 11-dimensional space. A supergravity theory is proposed.

Supersymmetry[1] is a new kind of symmetry between bosons and fermion. A dynamical system has supersymmetry if it is invariant under a " supersymmetric rotation " that changes fermions into bosons and vice versa. This requires the system possessing the same number of bosonic and fermionic degrees of freedom. Supergravity[2] is the local supersymmetry theory.

N = 1 supergravity in 4-dimensional space coupled with chiral supermultiplets and vector supermultiplets is very interested in recent years for the phenomenological unification theory[3]. In this theory, the symmetries of the low-energy physics are taken in by a Lie group, of which the chiral and vector supermultiplets are some adequate representations guaranteeing the theory anomly-free. To respect the symmetry of physics in a more fundamental way, there are two ways to go furthere, i.e., one could go beyond N = 1 to extended supersymmetry or beyond 4 dimensions to higher dimensions. But, it is regretful that the parity violation in low-energy phenomena suggests that physics is chiral and argues against extended supersymmetry[4]. Therefore, supergravity theory in higher dimensions is the promising candidate. Furthermore, in higher dimensional space, in the case of even dimensions, extended supergravity coupled with adequate supersymmetric Yang-Mills fields may be chiral when compactified to $d=4$, for example, we can mention the $d=6$, $N=2$ chiral anomaly-free supergravity interacting with $E(6) \times E(7) \times U(1)$ Yang-Mills field plus hypermultiplets[5].

E. Cremmer et al[6] constructed a supergravity theory in 11-dimensional space, the supermultiplet is $(e_M{}^A, \psi_M, A_{MNP})$ with (128+128) degrees of freedom, where $e_M{}^A$ is the vielbein field, ψ_M is the gravitino, A_{MNP} is a 3-form. The field content is simple and the theory is unique. The interesting thing is that this theory has a local SU(8) symmetry[7]. But, being in odd dimensional space, $d=11$ supergravity has no chiral fermion.

Does supergravity theory exist beyond 11 dimensions? Usually it is believed that 11 is the highest dimensions allowing supergravity to exist. In fact, a Majorana spinor in 11-dimensional space has 16 degrees of freedom, and correspondingly the theory has $N=8$ fermionic operators Q_i in 4 dimensions. The number 8 is the largest N to make the theory containing no particle with spin $J > 2$. Beyond 11 dimensions, the number N would be 16 or even more. Furthermore, the degrees of freedom of ψ_M are 128, 288, 320, 704 in 11, 12, 13, 14 dimensions respectively, but that of the vielbein and 3-form A_{MNP} are 44, 54, 65, 77, and 84, 120, 165, 220 respectively. It is obvious that the number of fermion degree of freedom increases much more rapidly than that of boson's. Therefore, beyond 12 dimensions, there must be much boson fields to match fermion fields. We conclude that 12-dimensional space is the only candidate allowing supergravity theory beyond 11 dimensions.

L. Castellani et al[8] studied supergravity theory in 12-dimensional space. They gave up the restriction that there is no particle having spin $J > 2$, and worked in a 12-dimensional space, of which the metric has signature (1,11), i.e. one has one time dimension and eleven space dimensions. The field content is $(e_M{}^A, A, A_M, A_{MN}, A_{MNPQ}, \psi_M, \lambda)$, where A is a scalar field, λ is a Majorana spinor field. They discovered that the theory is inconsistent.

We should look for other way, and this may be extra time dimension. Usually, extra dimentions in Kaluza-Klein theory are all space-like, this lead to positive energy in 4 dimensions. In recent years, however, extra time dimension are investigated[9], and it is believed that it is possible to get satisfactory 4-dimensional theory after compactification. So, we relax the signature of space. We have shown[10] that in the 12-dimensional space with signature (2,10) there exists Majorana-Weyl spinor having only 16 independent components just as a Majorana spinor in 11-dimensional space. We don't try to give the algebraic proof here but try to verify it with the following Majorana representation of gamma matrices:

$$
\begin{array}{llllllll}
\Gamma_1 & = & \sigma_2 \times 1 \times 1 \times 1 \times 1 \times 1 & \text{(antisymmetric)} \\
\Gamma_2 & = - & \sigma_1 \times \sigma_2 \times \sigma_1 \times 1 \times 1 \times 1 & \text{(antisymmetric)} \\
\Gamma_3 & = i & \sigma_1 \times \sigma_1 \times \sigma_1 \times 1 \times 1 \times 1 & \text{(symmetric)} \\
\Gamma_4 & = i & \sigma_1 \times \sigma_3 \times \sigma_1 \times 1 \times 1 \times 1 & \text{(symmetric)} \\
\Gamma_5 & = i & \sigma_1 \times 1 \times \sigma_2 \times \sigma_2 \times 1 \times 1 & \text{(symmetric)} \\
\Gamma_6 & = i & \sigma_1 \times 1 \times \sigma_3 \times 1 \times 1 \times 1 & \text{(symmetric)} \\
\Gamma_7 & = i & \sigma_1 \times 1 \times \sigma_2 \times \sigma_1 \times \sigma_2 \times \sigma_1 & \text{(symmetric)} \\
\Gamma_8 & = i & \sigma_1 \times 1 \times \sigma_2 \times \sigma_1 \times 1 \times \sigma_2 & \text{(symmetric)} \\
\Gamma_9 & = i & \sigma_1 \times 1 \times \sigma_2 \times \sigma_1 \times \sigma_2 \times \sigma_3 & \text{(symmetric)} \\
\Gamma_{10} & = i & \sigma_1 \times 1 \times \sigma_2 \times \sigma_3 \times \sigma_1 \times \sigma_2 & \text{(symmetric)} \\
\Gamma_{11} & = i & \sigma_1 \times 1 \times \sigma_2 \times \sigma_3 \times \sigma_2 \times 1 & \text{(symmetric)} \\
\Gamma_{12} & = i & \sigma_1 \times 1 \times \sigma_2 \times \sigma_3 \times \sigma_3 \times \sigma_2 & \text{(symmetric)} \\
\bar{\Gamma} & = & \sigma_3 \times 1 \times 1 \times 1 \times 1 \times 1 & \text{(real)} \\
C & = & 1 \times \sigma_2 \times \sigma_1 \times 1 \times 1 \times 1 & \text{(antisymmetric)}
\end{array} \tag{1}
$$

where σ_i are Pauli matrices, 1 is the 2x2 unity matrix, the $\bar{\Gamma}$ matrix and the charge-conjugation matrix C have the following properties:

$$\{\bar{\Gamma}, \Gamma^A\} = 0, \qquad \bar{\Gamma}^2 = 1; \tag{2}$$

$$C\Gamma^A C^{-1} = -(\Gamma^A)^T, \qquad C^\dagger C = 1, \qquad C^T = -C. \tag{3}$$

We can see fror (1) that the charge-conjugation matrix C commutes with the $\bar{\Gamma}$ matrix, and consequently with the chiral projection operators:

$$P_{R,L} = \frac{1}{2}(1 \pm \bar{\Gamma}) \tag{4}$$

also, henceforth, the Majorana condition and Weyl condition on a spinor are consistent with each other.

Since we have Majorana-Weyl condition on ψ_M, we have a supergravity theory with a (176+176) supermultiplet ($e_M{}^A$, A, B, A_{MNP}, ψ_M, λ) without the need of the 4-form A_{MNPQ}, where A, B are scalars, λ is a Majorana spinor. The action reads

$$I = \int d^{12}x\, \mathcal{L}$$

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{2}eR - \frac{i}{2}e\bar{\psi}_M\Gamma^{MNP}D_N\psi_P - \frac{1}{48}eF_{MNPQ}F^{MNPQ} \\
& -\frac{1}{2}ie\bar{\lambda}\not{D}\lambda - \frac{1}{2}e(\partial_M A)^2 - \frac{1}{2}e(\partial_M B)^2 \\
& + \frac{1}{192}e\left(\bar{\psi}_K\Gamma^{KLMNPQ}\psi_L + 12\bar{\psi}^M\Gamma^{NP}\psi^Q\right]F_{MNPQ} \\
& + \frac{\sqrt{2}}{96}ie\left[\bar{\lambda}\Gamma^{MNPQ}\Gamma^S\psi_S\right]F_{MNPQ} + \frac{e}{4}\bar{\lambda}\not{\partial}(A - i\bar{\Gamma}B)\Gamma^S\psi_S \\
& - \frac{1}{24^2}e(\bar{\lambda}\Gamma^{MNPQ}\lambda)F_{MNPQ} + \text{four-fermion terms},
\end{aligned} \tag{5}$$

where

$$F_{MNPQ} = 4\,\partial_{[M} A_{NPQ]} \,.$$

The action is invariant under the following supersymeetry transformation:

$$\begin{aligned}
\delta e_M^{\ A} &= \tfrac{1}{2} i \bar{\epsilon} \Gamma^A \psi_M, \quad \delta A = \bar{\epsilon}\lambda, \quad \delta B = i\bar{\epsilon}\bar{\Gamma}\lambda, \\
\delta\lambda &= -i \not{D}(A + i\bar{\Gamma}B)\epsilon + \tfrac{\sqrt{2}}{24}\Gamma^{MNPQ}F_{MNPQ}\,\epsilon, \\
\delta\psi_M &= D_M\epsilon + \tfrac{i}{288}(\Gamma^{PQRS}{}_M - 8\,\delta_M^P \Gamma^{QRS})\,F_{PQRS}\,\epsilon, \\
\delta A_{MNP} &= \bar{\epsilon}\Gamma_{MN}\psi_P + i\sqrt{2}\,\bar{\epsilon}\Gamma_{MNP}\lambda\,.
\end{aligned} \tag{6}$$

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